Multi-level μ-Finite Element Analysis for Human Bone Structures

Peter Arbenz[1], Harry van Lenthe[2], Uche Mennel[1], Ralph Müller[2], and Marzio Sala[1]

[1] Institute of Computational Science, ETH Zürich, 8092 Zürich, Switzerland
[2] Institute for Biomedical Engineering, ETH Zürich, 8092 Zürich, Switzerland

Impairment of the musculoskeletal system, e.g. osteoporosis or osteoarthrosis, are among the most frequent diseases in the industrialized world. Osteoporosis is a disease characterized by low bone mass and deterioration of bone microarchitecture. It leads to increased bone fragility and risk of fracture, particularly of the hip, spine and wrist. Worldwide, lifetime risk for osteoporotic fractures in women is estimated close to 40%; in men risk is 13% [3]; osteoporosis is second only to cardiovascular disease as a leading health care problem (World Health Organization). Osteoporotic fractures are a major cause of severe long-term pain and physical disability, and have an enormous impact on the individual, society and health care social systems.

With the advent of fast and powerful computers, simulation techniques are becoming popular for investigating the mechanical properties of bone. Using microstructural finite element (μFE) models generated directly from computer reconstructions of trabecular bone it is now possible to perform a 'virtual experiment', i.e. to simulate a mechanical test in great detail and with high precision.

Ideally, the development of a system with microstructural resolution better than 50 μm would allow in-vivo measurement of patients at different instances in time and at different anatomical sites. Unfortunately, such systems are not yet available, but the resolution at peripheral sites is approaching a level that allows elucidation of individual microstructural bone elements. Faster and more precise peripheral quantitative CT systems (pQCT) are now reaching the market, allowing for in-vivo patient measurements with an isotropic resolution better than 100 μm.

The basic mathematical model of the problem is given by the Lamé equations of elasticity. Their finite element discretization leads to a linear algebraic system

$$K\mathbf{u} = \mathbf{f}, \tag{*}$$

where K is the stiffness matrix. Since K is symmetric positive definite the preconditioned conjugate gradient method is applied for solving (*). In a similar approach as Adams et al. [1, 2] we employ a AMG multilevel preconditioner based on smoothed aggregation [5]. Such preconditioners are optimal and require relatively little memory. Operator complexities below 1.5 are common.

Our parallel C++ code is implemented with the Trilinos framework [4]. All tasks, i.e., distribution of the mesh data, load balancing, matrix assembly, the solution with the preconditioned conjugate gradient algorithm, and disk I/O are fully parallel. Numerical experiments with problem sizes up to 200 million degrees of freedom have been conducted on the 1100 processors Cray XT3 at the Swiss National Supercomputing Centre in Manno and on a smaller 596 processors Linux cluster at ETH Zürich.

Weak scalability tests with an artificial problem, a 'bone' that is extensible in all space dimension, proved that our algorithm scales very well. Simulation with real bones, distal radii of human forearms, are presented as well.

References

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