Efficient technique for solving low Mach number compressible multiphase problems

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Computation of unstationnary multifluid flows requires numerical strategies that are able to deal with very different physical regimes. The main difficulty, in this case, is to develop an efficient, accurate and robust method for large ratio of compressibility and density (gas/water), able to handle complex geometries (2D and 3D) with high grid aspect ratio, able to deal with material interfaces, ...

In this investigation, the fundamental model is described by a compressible Navier-Stokes system with a volume average description of the interface motion. This results in a system of hyperbolic partial differential equations which cannot be put in conservation form [2, 8]. This system writes formaly as

$$\frac{\partial W}{\partial t} + \operatorname{div} F(W) = B \cdot \nabla W \tag{1}$$

where W is the state variable (mass, momentum, total energy, volume fraction), F is the flux and $B \cdot \nabla W$ describes the non conservative effects. The equations contain entropy, vorticity, interfaces motion and acoustic modes at very different scales. If an explicit method is used, the maximum type step is a function of the mesh size h and the different wave speeds : material and entropy wave and the accoustic waves that are usually faster. The larger the maximum wave speed, the smaller the time step, thus this strategy is, in general, not efficient. The situation can be even worse when viscous effects are taken into account. This can be improved if preconditionning techniques associated with implicit schemes are used. The preconditionning technique we consider are of two types, either purely algebraic, or in the case of low Mach number flow, associated to physical considerations.

Our numerical method relies on a mixed finite volumes/finite elements formulation with an upwind Godunov type solver for the hyperbolic component of the model and a Galerkin approximation for the dissipative effects. Following a remark of [1, 6], the design of this method prevents numerical oscillations of pressure at contact discontinuities. In order to recover the correct asymptotic behavior at the low Mach limit, the pressure equation is relaxed with a time scale of the order of the square of the Mach number [7]. This preconditioning approach is simple, well adapted to multiphase flow and recovers the pioneer work of Turkel [9] and some more recent work [5].

During a time step of the implicit scheme, knowing W^n, the state variable W^{n+1} is computed in order to satisfy the following nonlinear system,

$$\mathcal{M}\frac{W^{n+1} - W^n}{\Delta t} + \Phi\left(W^{n+1}\right) = 0,$$

where $\mathcal{M}$ is the matrix of cells volumes, $\Phi\left(W^{n+1}\right)$ the discrete approximation of the spatial derivatives $\operatorname{div} F(W) - B \cdot \nabla W$ in (1). In practice, this nonlinear system is solved by a the resolution of linear systems given by

$$\left[\frac{1}{\Delta t}\mathcal{M} + \frac{\partial \Phi}{\partial W}\left(W^n\right)\right]\delta W = -\Phi\left(W^n\right)$$

where $\delta W = W^{n+1} - W^n$. Second order accurate schemes both in time and space can be performed by the defect correction method. It uses only first order Jacobian. In any case, we need to solve accurately the linear systems to make the result consistant with the original non linear form. At least the linear systems should be solved at the order $O(\Delta h)$ for first order accurate schemes and at order $O(\Delta h^2)$ for second order accurate schemes.

Therefore, numerical methods proposed with the assumption of hyperbolicity of the system becomes hill conditioned at this limit. As a consequence, the iterative methods used in the numerical algorithm implemented in the software FluidBox may have a worse convergence behavior. To solve 3D problems up to several millions of unknowns on a large number of processors, the parallelization of FluidBox relies on a domain decomposition, and a global preconditioning is unavoidable for performance efficiency in the context of parallel computing. Hence, a collaboration inside the INRIA ScAlApplix project has been setup to use the high performance solver library PaStiX [3, 4] that provides in particular parallel complete factorization algorithms on clusters of SMP nodes to solve large scale sparse systems. The parallel assembly algorithm has also been adapted to the data distribution induced by our parallel solver and allows a good load balancing in this context.

One of the aims of this presentation is to illustrate the advantages to use a solver based on a direct method for low Mach number compressible multiphase problems in the context of high performance parallel computations.

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