Aitken-Schwarz DDM to solve Darcy flow in heterogeneous underground mediums

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The objective of this work is to show the potentiality of the Aitken-Schwarz domain decomposition method to simulate the transport of fluids solutions in heterogeneous underground mediums. To study the phenomenology of the hydraulic properties, we use a stochastic model, where the data are random variables according to a prescribed distribution. The techniques of homogenization to model the studied heterogeneous mediums do not apply, because characteristic scales do not exist. All the computing scales need to be taken in account and lead to develop extensible and effective numerical methods to analyze the effects of scalings.

The physical modelling of the heterogeneous mediums leads to several types of difficulties even for the linear Darcy equation:

$$\begin{cases} \nabla.(k(x)\nabla h(x)) = f(x),\ x \in \Omega \subset R^3 \\ Bh(x) = g(x),\ x \in \partial\Omega \end{cases} \tag{1}$$

Where $h(x)$ represents the hydraulic head, and $k(x)$ the hydraulic conductivity of the media, B represents formally the boundary conditions. The treatment of various scales leads to solve sparse linear systems of very big size with bad condition number due to the heterogeneous permeability.

Classical domain decomposition methods such as generalized schwarz method can help to reduce the numerical complexity. But these methods have a slow convergence rate. Computing for optimal parameters for the transmission conditions at interfaces between subdomains is difficult when the hydraulic conductivity is randomly distributed inside the subdomains.

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At the opposite, the Aitken-Schwarz domain decomposition method [4,5] is a parameter free method. This method is based on the pure linear convergence of the Schwarz method in order to speed up the convergence of the Fourier modes of the solution at the interface toward those of the converged solution. This method is highly tolerant to low bandwidth and high latency and its scalability properties have been demonstrated on 1256 processors of 3 Cray T3E connected with a standard 5Mb/s ethernet network in the experiments described in [1], showing its competitivity in the case of $O(10)$ subdomains.

In case of non separable operators this Fourier modes of the solution at the artificial interfaces are coupled. Some adaptivity in the build of the Aitken matrix has been proposed in [3] in order to reduce the number of Fourier modes involved in the acceleration.

We will present the adaptative Aitken-Schwarz methodology and will compare the results obtained with parallel methods such as Hypre and / or PSPASES that have been applied on this problem [2].

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