A new cement to glue nonconforming grids with Robin interface conditions in the finite element case

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We consider an optimized Schwarz domain decomposition method, introduced in [3] (see [1] for the finite volume case), that allows for the use of Robin interface conditions on nonconforming grids. Such interface conditions have proven to be an efficient approach to domain decomposition methods in the case of conforming approximations (see [4]). On the other hand, using nonconforming grids allows for parallel generation of meshes, for local adaptative meshes and fast and independant solvers. The mortar method, first introduced in [2], enables the use of nonconforming grids. It is also well suited to the use of "Dirichlet-Neumann" or "Neumann-Neumann" preconditioned conjugate gradient method applied to the Schur complement matrix. But the mortar method can't be used easily with optimized interface conditions in the framework of Schwarz type methods.
We present the nonconforming approach proposed in [3], for the diffusion equation discretized by a finite element method. The nonconforming domain decomposition method is proved to be well posed, and the iterative solver to converge. The error analysis is performed. We give an efficient algorithm (see [3]) to perform the projection between arbitrary grids. We iterpret the method as a Jacobi algorithm applied to an interface problem. Then we replace the Jacobi algorithm by a Krylov type algorithm in order to accelerate the convergence. We present new numerical results that illustrate the method.

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