Parallel Multistage Stochastic Programming Model Used in Portfolio Management

Maria Lucka [a], Igor Melichercik [b], Ladislav Halada [c]

[a] *Institute for Software Science, University of Vienna, Nordbergstrasse 15/C/3, A-1090 Vienna, Austria*

[b] *Department of Applied Mathematics and Statistics, Faculty of Mathematics, Physics and Informatics, Comenius University, Mlynska Dolina, Bratislava, Slovakia*

[c] *Institute for Informatics, Slovak Academy of Sciences, Dubravska 9, 842 37 Bratislava, Slovakia*

Extended abstract

Problems of portfolio management can be viewed as multi-period dynamic decision problems. We present a model for allocation of financial resources to bond indices in different currencies. The future uncertainty is represented by stochastic scenario trees, where stochastic properties of possible future development of exchange rates and interest rates are incorporated. The objective is to maximize the expected value of the portfolio at a defined time horizon.

Mathematically such models lead to multistage stochastic programming. The probabilistic scenario trees are ”bushy” and the number of scenarios grows exponentially with the number of stages. We have developed and compared two-stage and three-stage models that were tested by using historical data of interest rates and exchange rates. The portfolios were composed of 4 indices of government bonds in different currencies (USD, EUR, GBP and CHF). The scenarios of future development of interest rates and exchange rates were generated using Monte Carlo simulations.

Mathematical formulation of the problem leads to solving large linear systems of equations, where the matrix of the system is regular and sparse. For solving the multi-stage stochastic programming model the Interior Point Method (IPM) was used. The IPM is solved according to the Mehrotra’s Predictor Corrector algorithm (MPC) algorithm, as it is defined in [1]. The MPC creates iteratively a sequence of solutions, whereby the crucial step is solving the system of linear equations with the system matrix $A^{(3)}D^{(3)}(A^{(3)})^T$ in the case of the three-stage model, where $D^{(3)}$ is a diagonal matrix, and $A^{(3)}$ is a matrix coming from the three-stage program-

ming model. The application of the BQ [2] allows a decomposition of the matrix $A^{(3)}$ to smaller linear systems. This allows efficient parallelization and diminish the memory requirements. The small linear systems correspond in general to two-stage stochastic problems. For solving these, the linear algebra package LAPACK [3]is used.

The designed parallel algorithm is written in the Fortran/MPI programming language. The experiments were executed on a IBM Linux cluster, University of Technology, Vienna.

The aim of the paper is:

- to formulate a model for allocation of financial resources
- to test and compare two-stage and three-stage parallel programming model from the point of view of quality
- to present the time measurement results of the parallelization on cluster

The achieved results have shown that it makes sense to increase the number of stages, especially in case of a good feeling of future market development.

References

[1] Stephen. J. Wright, *Primal-Dual Interior Point Methods*, ISBN 0-89871-382-X, SIAM, 1997.

[2] J. R. BIRGE AND L. QI, *Computing block-angular Karmarkar projections with applications to stochastic programming*, Management Sci. 34, 1988, pp. 1472–1479.

[3] E. ANDERSON, Z. BAI, C. H. BISCHOF, S. BLACKFORD, J. W. DEMMEL, J. J. DONGARRA, J. DU CROZ, A. GREENBAUM, S. HAMMARLING, A. MCKENNEY, AND D. C. SORENSEN, LAPACK *Users' Guide*, 3rd edn. SIAM Press, Philadelphia PA, 1999.

[4] L. Halada, M. Lucka and I. Melichercik, Optimal Multistage Portfolio Management Using a Parallel Interior Point Method,*Proc. of ALGORITMY'2005*, High Tatras, Slovakia, March 13-18, pp. 359-368, 2005.

[5] M. Holmer, R. Mc Kendall, Ch. Vassiadou-Zeniou and S. A. Zenios, Dynamic Models for Fixed-Income Portfolio Management under Uncertainty, *Journal of Economic Dynamics and Control* **22**, pp.1517–1541, 1998.

[6] L. Halada, S. Benkner, M. Lucka, Parallel Birge and Qi Factorization of Three-stage Stochastic Linear Programs, *Parallel and Distributed Computing Practices* **5**, No.3, Nova Science Publishers, pp. 301–311, 2002.

[7] G. CH. Pflug, L. Halada, A Note on the Recursive and Parallel Structures of the Birge and Qi Factorization for Tree Structured Linear Programs, *Computational Optimization and Applications* **24**, No. 2–3, pp. 251–265, 2003.