Combining Building Blocks for Parallel Multilevel Matrix Multiplication

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Outline

Introduction

- matrix multiplication is one of the **core computations** in many algorithms from scientific computing;
  $\rightarrow$ many efficient realizations have been invented;
- efficient **sequential** implementations: ATLAS, PHiPAC;
  efficient **parallel** implementations: SUMMA, PDGEMM;
- execution platforms differ in important characteristics **memory hierarchy, communication architecture**
  efficient implementation must exploit these characteristics;
  $\rightarrow$ **adaptivity** is an important property;

Introduction

- **multiprocessor task** (M-task) implementations have been successful for many platforms for different application areas;
  advantage: reduction in communication time, better scalability;
- **approach:** M-task implementation of matrix multiplication;
- **goal:** competetiveness with ScaLAPACK and vendor-specific implementations;

Outline

Specific Approach

- **hierarchical M-task implementation** with combination of different algorithms to exploit specific characteristics of the execution platform;
- **lower level:** efficient adaptive and/or highly tuned **sequential algorithm** to use single-processor architecture;
- **upper level:** hierarchical (recursive) algorithm to improve communication behavior → **Strassen** algorithm;
- **intermediate level:** hierarchical (recursive) algorithm **tpmm** to improve memory access and communication behavior for multi-processor nodes;

Specific Approach

result: multi-level algorithm which allows adaptions to different execution platforms:

- different **combinations** of algorithms;
- different **levels of recursions** of upper level Strassen algorithm;
- different lower level **sequential algorithms**;
- different **block sizes** for low-level algorithms;
- different **schedulings** for upper level Strassen algorithm;

Outline

Execution environment: Tlib

- The **runtime library Tlib** allows the realization of **hierarchically structured M-tasks** programs;
- Tlib is based on **MPI** and allows the use of arbitrary MPI functions.
- The Tlib API provides support for
  - the **creation** and **administration** of a **dynamic hierarchy of processor groups**;
  - the **coordination** and **mapping** of nested M-tasks;
  - the **handling** and **termination** of recursive calls and group splittings;
  - the organization of **communication between M-tasks**;
- The splitting and mapping can be adapted to the execution platform.

Interface of the Tlib library

Type of all **Tlib tasks** (basic and coordination):

```
void *F (void * arg, MPI_Comm comm, T_Descr *pdescr)
```

- `void * arg` packed arguments for `F`;
- `comm` MPI communicator for internal communication;
- `pdescr` pointer to a descriptor of executing processor group;

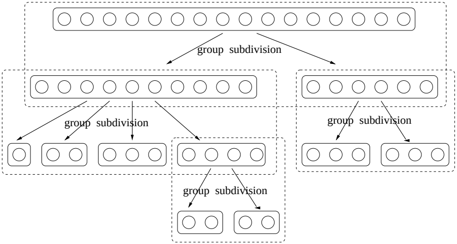
The function `F` may contain calls of Tlib functions to **create sub-groups** and to **map sub-computations** to sub-groups.

Splitting operations for processor groups

```
int T_SplitGrp( T_Descr * pdescr,
                T_Descr * pdescr1,
                float per1,
                float per2)
```

- pdescr: current group descriptor
- pdescr1: new group descriptor for two new subgroups
- per1 relative size of first subgroup
- per2 relative size of second subgroup

```
int T_SplitGrpParfor( int n,
                      T_Descr *pdescr,
                      T_Descr *pdescr1,
                      float p[])
```




```
int T_SplitGrpExpl( T_Descr * pdescr,
                    T_Descr * pdescr1,
                    int color,
                    int key)
```

Concurrent execution of M-tasks

Mapping of two concurrent M-tasks to processor groups:

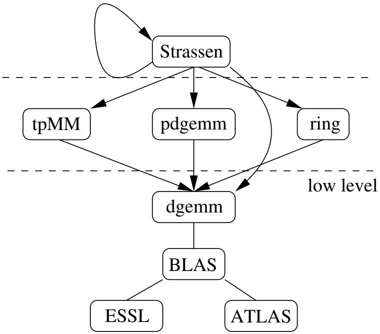
```
int T_Par( void * (*f1)(void *, MPI_Comm, T_Descr *),
           void * parg1, void * pres1,
           void * (*f2)(void *, MPI_Comm, T_Descr *),
           void * parg2, void * pres2,
           T_Descr *pdescr)
```

- `f1` function for first M-Task
- `parg1`, `pres1` packed arguments and results for `f1`
- `f2` function for second M-Task
- `parg2`, `pres2` packed arguments and results for `f2`
- `pdescr` pointer to current group descriptor describing two groups

Outline

Algorithm combinations

- A **combination** of different algorithms for matrix multiplication can lead to very competitive implementations:
  Strassen method, tpMM, Atlas;
- Illustration:



Recursive splitting – Strassen algorithm

- **matrix multiplication** $C = AB$ with $A, B, C \in I\!R^{n \times n}$:
  decompose A, B into square blocks of size $n/2$:

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

- compute the **submatrices** $C_{11}, C_{12}, C_{21}, C_{22}$ separately according to

$$\begin{array}{ll} C_{11} = Q_1 + Q_4 - Q_5 + Q_7 & C_{12} = Q_3 + Q_5 \\ C_{21} = Q_2 + Q_4 & C_{22} = Q_1 + Q_3 - Q_2 + Q_6 \end{array}$$

- compute $Q_1, \ldots, Q_7$ by **recursive calls**:

$$\begin{array}{lll} Q_1 & = & strassen(A_{11} + A_{22}, B_{11} + B_{22}); \\ Q_2 & = & strassen(A_{21} + A_{22}, B_{11}); \\ Q_3 & = & strassen(A_{11}, B_{12} - B_{22}); \\ Q_4 & = & strassen(A_{22}, B_{21} - B_{11}); \end{array} \qquad \begin{array}{lll} Q_5 & = & strassen(A_{11} + A_{12}, B_{22}); \\ Q_6 & = & strassen(A_{21} - A_{11}, B_{11} + B_{12}); \\ Q_7 & = & strassen(A_{12} - A_{22}, B_{21} + B_{22}); \end{array}$$

Strassen algorithm – task scheduling

- organization into four tasks
  `Task_C11()`, `Task_C12()`, `Task_C21()`, `Task_C22()`:

Task C11 on group 0	Task C12 on group 1	Task C21 on group 2	Task C22 on group 3
compute Q_1	compute Q_3	compute Q_2	compute Q_1
compute Q_7	compute Q_5	compute Q_4	compute Q_6
receive Q_5	send Q_5	send Q_2	receive Q_2
receive Q_4	send Q_3	send Q_4	receive Q_3
compute C_{11}	compute C_{12}	compute C_{21}	compute C_{22}

- realization with Tlib: `Task_C11()` and `Task_q1()`

Strassen algorithm – main Tlib coordination function

```
void * Strassen (void * arg, MPI_Comm comm,
                 T_Descr *pdescr){
   T_Descr descr1;
   void * (* f[4])(), *pres[4], *parg[4];
   float per[4];

   per[0]=0.25; per[1]=0.25; per[2]=0.25; per[3]=0.25;
   T_Split_GrpParfor (4, pdescr, &descr1, per);
   f[0] = Task_C11; f[1] = Task_C12;
   f[2] = Task_C21; f[3] = Task_C22;
   parg[0] = parg[1] = parg[2] = parg[3] = arg;
   T_Parfor (f, parg, pres, &descr1);
   assemble_matrix (pres[0], pres[1], pres[2], pres[3]);
}
```

Strassen algorithm – Task_C11()

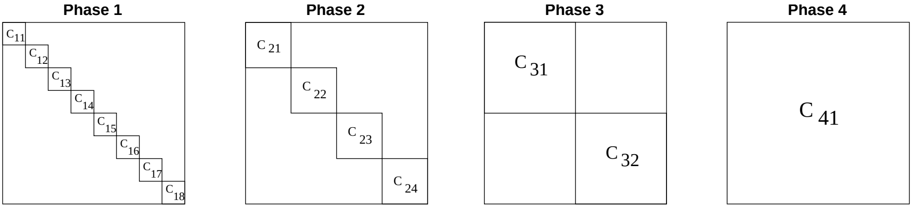
```
void * Task_C11 (void * arg, MPI_Comm comm,
                 T_Descr *pdescr){
   double **q1, **q4, **q5, **q7, **res;
   int i,j,n2;

   /* extract arguments from arg including matrix size n */
   n2 = n/2;
   q1 = Task_q1 (arg, comm, pdescr);
   q7 = Task_q7 (arg, comm, pdescr);
   /* allocate res, q4 and q5 as matrices of size n/2 times n/2 */
   /* receive q4 from group 2 and q5 from group 1 using parent comm. */
   for (i=0; i < n2; i++)
    for (j=0; j < n2; j++)
     res[i][j] = q1[i][j]+q4[i][j]-q5[i][j]+q7[i][j];
   return res;
}
```

Strassen algorithm – Task_q1()

```
void *Task_q1(void *arg, MPI_Comm comm, T_Descr *pdescr){
 double **res, **q11, **q12, **q1;
 struct struct_MM_mul *mm, *arg1;
 *mm = (struct_MM_mul *) arg;
 n2 = (*mm).n /2;
 /* allocate q1, q11, q12 as matrices of size n2 times n2 */
 for (i=0; i < n2; i++)
  for (j=0; j < n2; j++) {
   q11[i][j] = (*mm).a[i][j]+(*mm).a[i+n2][j+n2]; /* A11+A22 */
   q12[i][j] = (*mm).b[i][j]+(*mm).b[i+n2][j+n2]; /* B11+B22 */
  }
 arg1 = (struct_MM_mul *) malloc (sizeof(struct_MM_mul));
 (*arg1).a = q11; (*arg1).b = q12; (*arg1).n = n2;
 q1 = Strassen (arg1, comm, pdescr);
 return q1;
}
```

tpmm for memory hierarchies

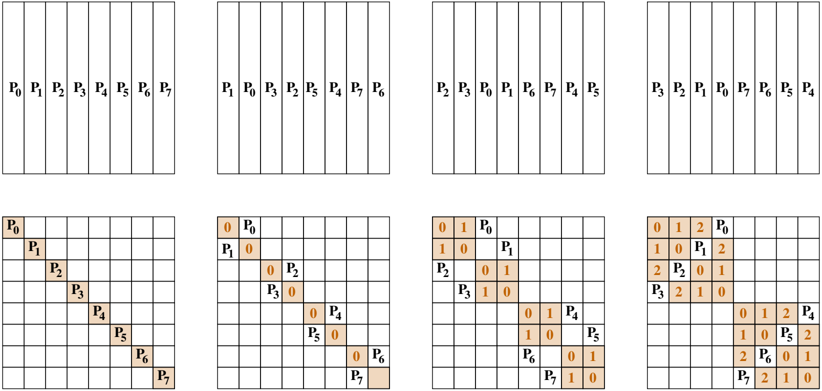
block structure of tpmm: $p = 2^i$ processors



tpMM(n,p) =
  for ($l = 1$ to $\log p + 1$)
    for $k = 1, \ldots, p/2^{l-1}$ compute in parallel
      compute_block (C_{lk});

tpmm for memory hierarchies

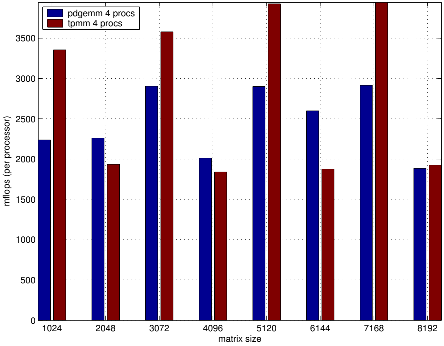
tpmm: data distribution for matrix B and computation order for 8 processors:

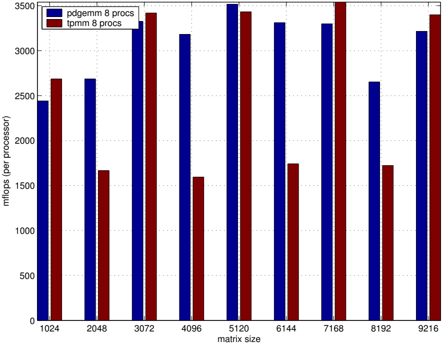


bottom level: **Atlas** for one-processor matrix multiplication

tpmm performance in isolation

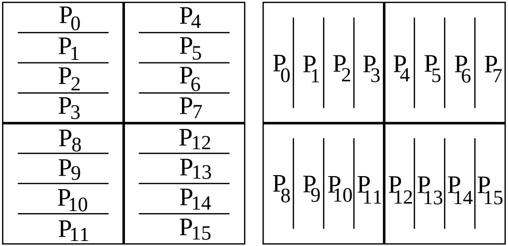
MFLOPS per processor for 4 and 8 processors on IBM Regatta:





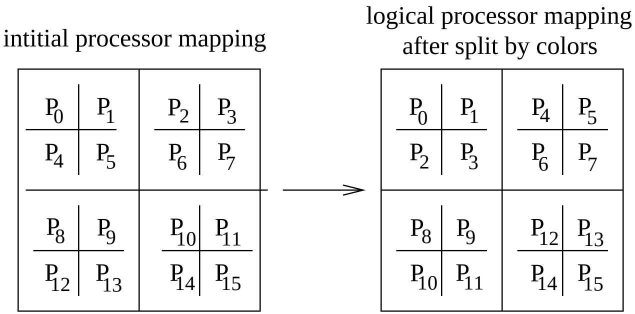
Combining Strassen and tpMM

- $p = 4^i 2^j$ processors with $i \geq 1$ and $j \geq 0$:
  4 processor groups are build at each recursion level of Strassen;
  2^j processors are used for the execution of tpmm;
- data layout for 16 processors:



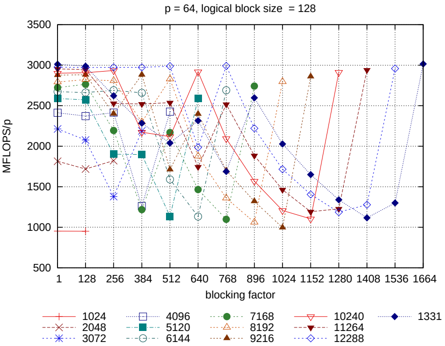
Combining Strassen and PDGEMM

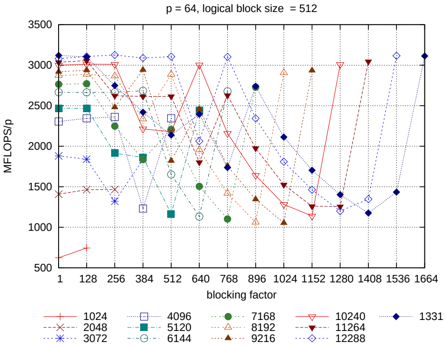
- $p = 4^i \cdot p = 2^d$ processors with $i \geq 1$ and $d \in \{0, 1\}$;
- The processors are mapped row-blockwise onto the blocks of A, B and C so that the blocks have size $\frac{n}{r} \times \frac{n}{c}$.
- data layout for 16 processors:



Tuning of PDGEMM

- impact of blocking factor on PDGEMM performance:

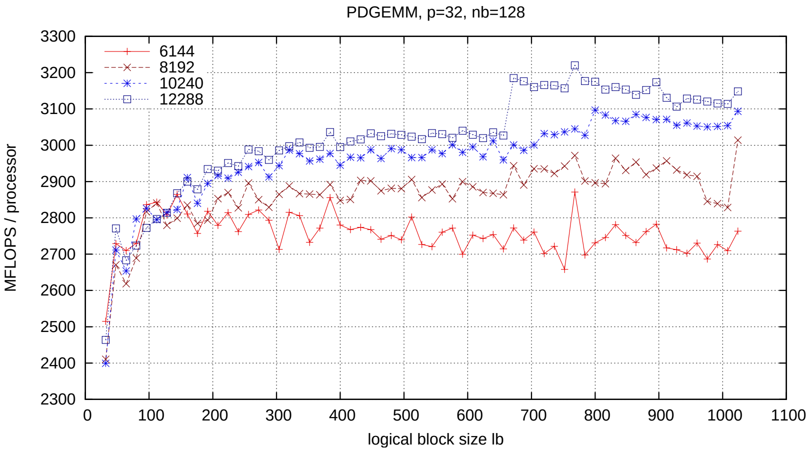




Opteron cluster with Infiniband interconnect

Tuning of PDGEMM

- impact of logical block size on PDGEMM performance:

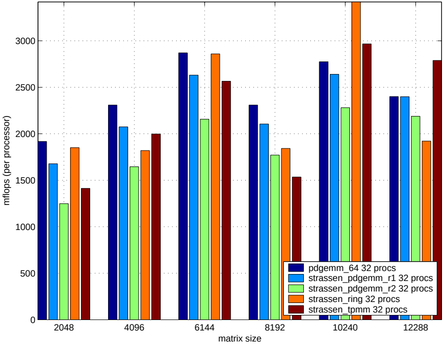


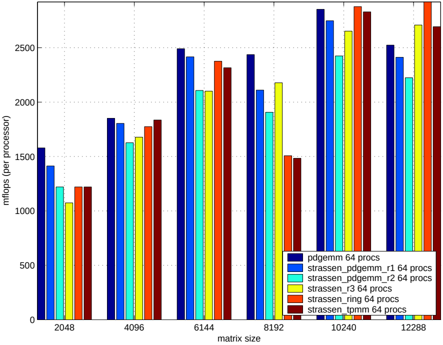
Opteron cluster with Infiniband interconnect

Outline

Experimental evaluation: IBM Regatta p690

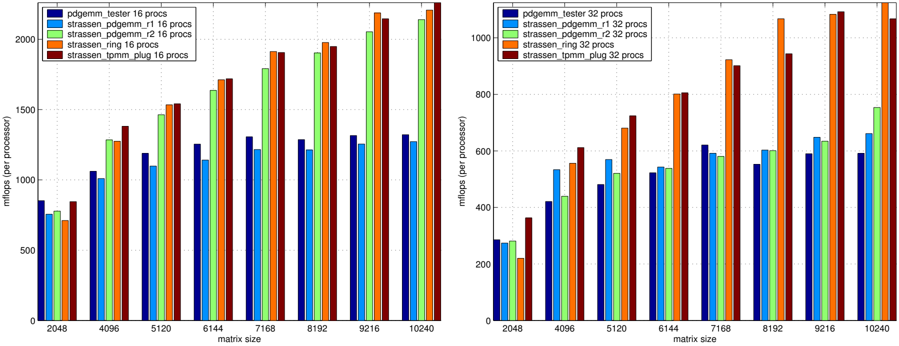
$p = 32$ and $p = 64$: MFLOPS for different algorithmic combinations:





Experimental evaluation: Dual Xeon Cluster 3 GHz

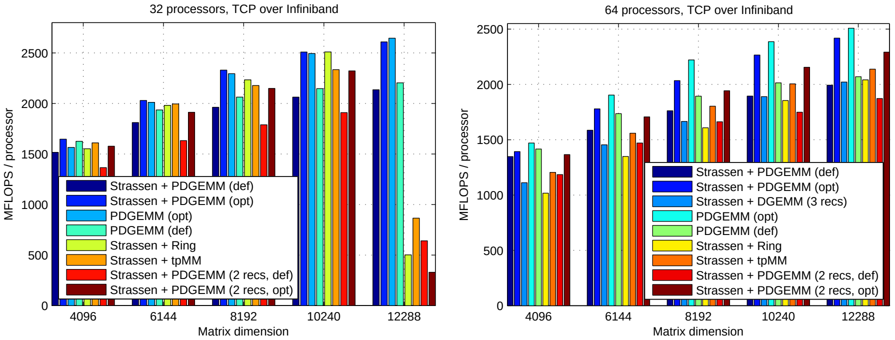
MFLOPS for different algorithmic combinations:



MFLOPS per processor for 16 and 32 processors

Experimental evaluation: Infiniband Cluster

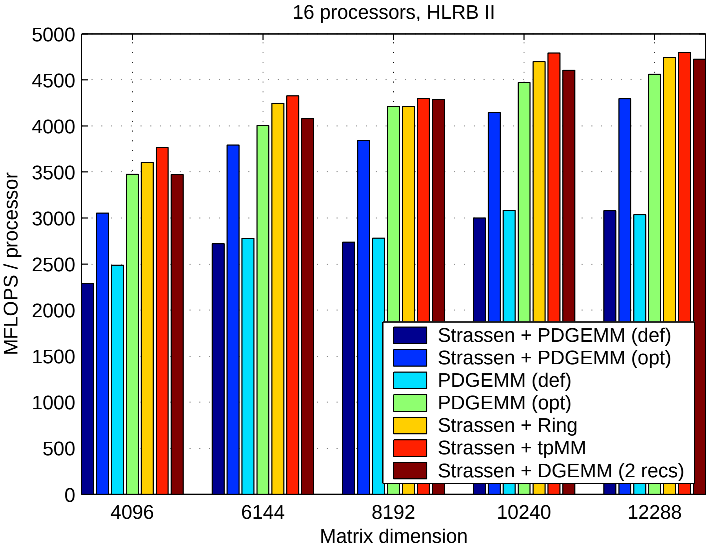
MFLOPS for different algorithmic combinations:



MFLOPS per processor for 32 and 64 processors

Experimental evaluation: SGI Altix

$p = 16$: MFLOPS for different algorithmic combinations:



Outline

Conclusions

- The combination of different algorithms as **building blocks** leads to competetive parallel implementations;
- There are many different ways to combine the building blocks; for different platforms, **different combinations lead to the best performance**;
- **automatic support** for finding the best combination is useful;
- outlook: the combination of different algorithms might be especially useful for **heterogeneous platforms**;
- further information:
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